Grade 10
Chapter 4

Quadratic Equations

- The general form of quadratic equation in the variable '*x*' is $ax^2 + bx + c = 0$, where *a*, *b*, *c* are real numbers and $a \neq 0$.
  For example, $3x^2 + 6x + 2 = 0,\ x^2 - 2 = 0$

- A real number '*k*' is said to be the root of the quadratic equation, $ax^2 + bx + c = 0$, if $ak^2 + bk + c = 0$
  For example, 3 and –10 are the roots of the quadratic equation, $x^2 + 7x - 30 = 0$, because
  $3^2 + 7 \times 3 - 30 = 9 + 21 - 30 = 30 - 30 = 0 =$ R.H.S.
  $(-10)^2 + 7 \times (-10) - 30 = 100 - 70 - 30 = 0 =$ R.H.S.

  Note: $x = \alpha$ (α may or may not be real) is a solution of the quadratic equation, $ax^2 + bx + c = 0$, if it satisfies the quadratic equation.

Solution of Quadratic Equation by Factorization Method

If we can factorize $ax^2 + bx + c = 0$, where $a \neq 0$, into a product of two linear factors, then the roots of this quadratic equation can be calculated by equating each factor to zero.

Example: Find the roots of the equation, $2x^2 - 7\sqrt{3}x + 15 = 0$, by factorisation.

Solution:

$2x^2 - 7\sqrt{3}x + 15 = 0$

$\Rightarrow 2x^2 - 2\sqrt{3}x - 5\sqrt{3}x + 15 = 0$

$\Rightarrow 2x(x - \sqrt{3}) - 5\sqrt{3}(x - \sqrt{3}) = 0$

$\Rightarrow (x - \sqrt{3})(2x - 5\sqrt{3}) = 0$

$(x - \sqrt{3}) = 0$ or $(2x - 5\sqrt{3}) = 0$

$\Rightarrow x = \sqrt{3}$ or $x = \frac{5\sqrt{3}}{2}$

Therefore, $\sqrt{3}$ and $\frac{5\sqrt{3}}{2}$ are the roots of the given quadratic equation.

Solution of Quadratic Equation by completing the square

A quadratic equation can also be solved by the method of completing the square.

Example: Find the roots of the quadratic equation, $5x^2+7x-6=0$, by the method of completing the square.

Solution:

$$5x^2+7x-6=0$$

$$\Rightarrow 5\left[x^2+\frac{7}{5}x-\frac{6}{5}\right]=0$$

$$\Rightarrow x^2+2\cdot x\cdot\frac{7}{10}+\left(\frac{7}{10}\right)^2-\left(\frac{7}{10}\right)^2-\frac{6}{5}=0$$

$$\Rightarrow \left(x+\frac{7}{10}\right)^2-\frac{49}{100}-\frac{6}{5}=0$$

$$\Rightarrow \left(x+\frac{7}{10}\right)^2=\frac{169}{100}$$

$$\Rightarrow \left(x+\frac{7}{10}\right)=\pm\sqrt{\frac{169}{100}}=\pm\frac{13}{10}$$

$$x+\frac{7}{10}=\frac{13}{10} \text{ or } x+\frac{7}{10}=\frac{-13}{10}$$

$$\Rightarrow x=\frac{13}{10}-\frac{7}{10}=\frac{3}{5} \text{ or } x=\frac{-13}{10}-\frac{7}{10}=-2$$

Therefore, –2 and $\frac{3}{5}$ are the roots of the given quadratic equation.

Quadratic Formula

The roots of the quadratic equation, $ax^2+bx+c=0$, are given by,

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, where $b^2-4ac\geq 0$

Example: Find the roots of the equation, $2x^2-3x-44=0$, if they exist, using the quadratic formula.

Solution:

$$2x^2-3x-44=0$$

Here, a = 2, b = –3, c = –4

$$\therefore b^2-4ac=(-3)^2-4\times 2\times(-44)=9+352=361>0$$

The roots of the given equation are given by $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

$\Rightarrow x = \frac{-(-3) \pm \sqrt{361}}{2 \times 2} = \frac{3 \pm 19}{4}$

$\Rightarrow x = \frac{3+19}{4} = \frac{11}{2}$ or $x = \frac{3-19}{4} = -4$

$\therefore$ The roots are –4 and $\frac{11}{2}$.

Nature of roots of Quadratic Equation

For the quadratic equation, $ax^2 + bx + c = 0$, where $a \neq 0$, the discriminant 'D' is defined as $\text{D} = b^2 - 4ac$

- The quadratic equation, $ax^2 + bx + c = 0$, where $a \neq 0$, has

(i) two distinct real roots, if $\text{D} = b^2 - 4ac > 0$

(ii) two equal real roots, if $\text{D} = b^2 - 4ac = 0$

(iii) no real roots, if $\text{D} = b^2 - 4ac < 0$

Example: Determine the nature of the roots of the equation, $2x^2 + 5x - 117 = 0$.

Solution:

Here, $a = 2$, $b = 5$, $c = -117$

$\therefore \text{D} = b^2 - 4ac = 5^2 - 4 \times 2 \times (-117) = 25 + 936 = 961 > 0$

Therefore, the roots of the given equation are real and distinct.