Must Cover Questions for JEE Advanced 2015 with Solutions

Maths>

Q1. IIT JEE 2012 Paper - 2

The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\ dx$ is

(a) 0

(b) $\frac{\pi^2}{2}-4$

(c) $\frac{\pi^2}{2}+4$

(d) $\frac{\pi^2}{2}$

Correct Answer: Option B

Let I = $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\ dx$.

$\Rightarrow$ I = $\int_{-\pi/2}^{\pi/2}x^2\cos x dx+\int_{-\pi/2}^{\pi/2}\left(\ln\frac{\pi+x}{\pi-x}\right)\cos x\ dx$

$\Rightarrow$ I $= \int_{-\pi/2}^{\pi/2}x^2\cos x dx$ + 0 $\left(\because \ln\frac{\pi+x}{\pi-x}\text{ is an odd function}\right)$

The function $x^2\cos x$ is an odd function. Therefore,

$$I = 2\int_0^{\frac{\pi}{2}} x^2 \cos x\,dx$$

$$= 2\left[x^2\sin x - 2(-x\cos x + \sin x)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi^2}{2} - 4$$

$$\therefore \int_{-\pi/2}^{\pi/2}\left(x^2 + \ln\frac{\pi+x}{\pi-x}\right)\cos x\ dx = \frac{\pi^2}{2} - 4$$

Hence, the correct option is B.

Q2. IIT JEE 2012 Paper - 1

If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a}-\vec{b}\right|^2 + \left|\vec{b}-\vec{c}\right|^2 + \left|\vec{c}-\vec{a}\right|^2 = 9$, then $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$ is

Correct Answer: 3

Solution:

Given, $\left|\vec{a}-\vec{b}\right|^2 + \left|\vec{b}-\vec{c}\right|^2 + \left|\vec{c}-\vec{a}\right|^2 = 9$ and $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$.

So,

$$2\left(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2\right) - 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{a}.\vec{c}) = 9$$

$$\Rightarrow 6 - 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 9 \quad \text{... (1)}$$

Now,

$$3 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{a}.\vec{c}) = |\vec{a} + \vec{b} + \vec{c}|^2 \quad \text{... (2)}$$

From (1) and(2), we get

$|\vec{a}+\vec{b}+\vec{c}|=0$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \vec{a}=-\vec{b}-\vec{c}$

Therefore,

$$\left|2\vec{a}+5\left(\vec{b}+\vec{c}\right)\right|=\left|2\vec{a}-5\vec{a}\right|$$
$$=3$$

Hence, the value of $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$ is 3.

Q3. JEE Advanced 2013 Paper - 2

Let ω be a complex cube root of unity with ω ≠ 1 and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n =

(a) 57

(b) 55

(c) 58

(d) 56

Solution:

Given, $P=\left[p_{ij}\right]_{1\times1}$ and $p_{ij}=\omega^{i+j}$.

For n = 1:

$$P=\left[\omega^2\right]_{1\times1}$$

Thus, we get

$$P^2=\left[\omega^2\right]\times\left[\omega^2\right]=\left[\omega^4\right]\neq0$$

For n = 2:

$$P=\begin{bmatrix}\omega^2 & \omega^3\\ \omega^3 & \omega^4\end{bmatrix}_{2\times2}=\begin{bmatrix}\omega^2 & 1\\ 1 & \omega\end{bmatrix}_{2\times2}\quad\left(\because\omega^3=1\right)$$

Thus, we get

$$P^2$$

$$= \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix} \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}$$

$$= \begin{bmatrix} \omega^4 + 1 & \omega^2 + \omega \\ \omega^2 + \omega & 1 + \omega^2 \end{bmatrix}$$

$$= \begin{bmatrix} \omega + 1 & \omega^2 + \omega \\ \omega^2 + \omega & 1 + \omega^2 \end{bmatrix}$$

$$= \begin{bmatrix} -\omega^2 & -1 \\ -1 & -\omega \end{bmatrix} \qquad \left(\because 1 + \omega + \omega^2 = 0\right)$$

$$\neq 0$$

For n = 3:

$$P = \begin{bmatrix} \omega^2 & \omega^3 & \omega^4 \\ \omega^3 & \omega^4 & \omega^5 \\ \omega^4 & \omega^5 & \omega^6 \end{bmatrix} = \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} \quad \left(\because \omega^3 = 1\right)$$

Thus, we get

$$P^2 = \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} \times \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \omega^4+1+\omega^2 & \omega^2+\omega+\omega^3 & \omega^3+\omega^2+\omega \\ \omega^2+\omega+\omega^3 & 1+\omega^2+\omega^4 & \omega+\omega^3+\omega^2 \\ \omega^3+\omega^2+\omega & \omega+\omega^3+\omega^2 & \omega^2+\omega^4+1 \end{bmatrix}$$

$$= \begin{bmatrix} \omega+1+\omega^2 & \omega^2+\omega+1 & 1+\omega^2+\omega \\ \omega^2+\omega+1 & 1+\omega^2+\omega & \omega+1+\omega^2 \\ 1+\omega^2+\omega & \omega+1+\omega^2 & \omega^2+\omega+1 \end{bmatrix} \qquad \left[\omega^3 = 1\right]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \left[1+\omega+\omega^2 = 0\right]$$

$$= 0$$

We observe that, $P^2 \neq 0$ for $n = 1$, $n = 2$ and $n = 3$ i.e. $P^2 \neq 0$ when n is not a multiple of 3.
We also observe tha, the numbers in options B, C and D are not multiple of 3.

Hence, the correct options are B, C and D.

Q4. JEE Advanced 2013 Paper - 1
A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224.
If the smaller of the numbers on the removed cards is k, then $k - 20$ =

Correct Answer: 5
Solution:

There are 1 to n cards.
Let the two cards removed from n cards be having numbers k and $k + 1$.

Sum of the numbers from 1 to n = $\frac{n(n+1)}{2}$.

Sum of the numbers after removing two cards = 1224

$\Rightarrow \frac{n(n+1)}{2}$ = 1224 + k + (k + 1)

$\Rightarrow n^2 + n - 4k = 2450$

$\Rightarrow n^2 + n - 2450 = 4k$

$\Rightarrow (n + 50)(n - 49) = 4k$

$\Rightarrow n > 49$

For, n = 50, $\frac{n(n+1)}{2}$ = 1275 (> 1224)

Also, for n = 50, $k = \frac{(50+50)(50-49)}{4} = 25$

Thus, $k - 20 = 5$

Q5. IIT JEE 2011 Paper - 1

Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then

(a) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(b) a focus of the hyperbola is (2, 0)

(c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

Solution:

The equation $x^2 + 4y^2 = 4$ can be written as,

$\frac{x^2}{4} + \frac{y^2}{1} = 1$,

Here, $A^2 = 4, B^2 = 1$.

$\therefore$ Eccentricity of the ellipse = $\sqrt{1-\frac{B^2}{A^2}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$

Thus, the eccentricity of the hyperbola is $\frac{2}{\sqrt{3}}$.

So, the focus of the ellipse is $(\pm Ae,0)=\left(\pm\sqrt{3},0\right)$.

It is given that, the hyperbola passes through the focus of the ellipse.

$\frac{3}{a^2}-0=1\Rightarrow a^2=3$

Now,

$b^2=a^2\left(e'^2-1\right)$

$\Rightarrow b^2=3\left(\frac{4}{3}-1\right)=1$

Therefore, equation of hyperbola is,

$\frac{x^2}{3}-\frac{y^2}{1}=1$

$\Rightarrow x^2-3y^2=3$

Focus of the hyperbola $(\pm ae,0)=\left(\pm\sqrt{3}\times\frac{2}{\sqrt{3}},0\right)=(\pm 2,0)$

Hence, the correct options are B and D.

Q6. IIT JEE 2011 Paper - 1

Let M and N be two 3×3 non − singular skew − symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2\left(M^TN\right)^{-1}\left(MN^{-1}\right)T$ is equal to

(a) M^2

(b) $-N^2$

(c) $-M^2$
(d) MN

Correct Answer: Option C
Given, MN = NM.
Since, M and N are skew-symmetric, therefore

$$M^T = -M$$

$$N^T = -N$$

Now,

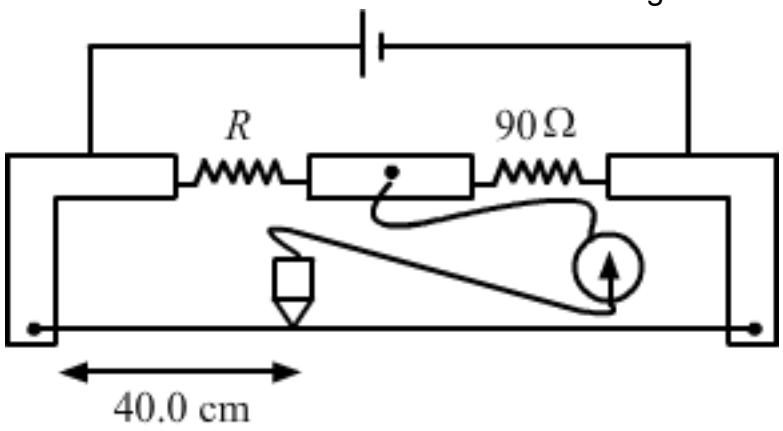
$$\begin{aligned}
&M^2N^2\left(M^TN\right)^{-1}\left(MN^{-1}\right)^T \\
&= M^2N^2(-MN)^{-1}\left(N^{-1}\right)^TM^T \\
&= M^2N^2(-MN)^{-1}\left(N^T\right)^{-1}(-M) \\
&= M^2N^2(MN)^{-1}(-N)^{-1}(M) \\
&= -M^2N^2N^{-1}M^{-1}N^{-1}M \\
&= -MMNM^{-1}N^{-1}M \\
&= -MNMM^{-1}N^{-1}M \quad (\because MN = NM) \\
&= -MNN^{-1}M = -M^2
\end{aligned}$$

Hence, the correct option is C.

Physics>

Q1. JEE Advanced 2014 Paper - 2

During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is



(A) 60 ± 0.15 Ω
(B) 135 ± 0.56 Ω
(C) 60 ± 0.25 Ω
(D) 135 ± 0.23 Ω

Correct Answer: Option C

The null point on the metre bridge is calculated by the following formula:

$$\frac{R_1}{R_2} = \frac{l}{(100-l)}$$

Here, $R_1 = R$ (unknown resistance)
$R_2 = 90\ \Omega$

$$\therefore \quad R = \frac{90l}{(100-l)} \quad ...(1)$$

Let the error in the calculation of the resistance R be ΔR.
Relative error is given by:

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} + \frac{\Delta l}{100-l}$$

$$\Rightarrow \frac{\Delta R}{60} = \frac{0.1}{40} + \frac{0.1}{60}$$

$$\Rightarrow \Delta R = \frac{0.5}{2} = 0.25$$

Putting l = 40 cm in equation (1), we have:
The value of the unknown resistance is given by

$$R = \frac{90 \times 40}{60} = 60\ \Omega$$

Hence, the value of the resistance can be written as $R = (60 \pm 0.25)\ \Omega$.

Hence, the correct option is C.

Q2. JEE Advanced 2013 Paper - 2

Match List I with List II and select the correct answer using the codes given below the lists:

	List I		List II
P.	Boltzmann constant	**1.**	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	**2.**	$[ML^{-1}T^{-1}]$
R.	Planck constant	**3.**	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	**4.**	$[ML^2T^{-2}K^{-1}]$

Codes:
P Q R S
(a) 3 1 2 4
(b) 3 2 1 4
(c) 4 2 1 3
(d) 4 1 2 3

Solution:

P,

$$\text{Use, } U = \frac{1}{2}kT$$

$$[U] = [k][T]$$

$$[ML^2T^{-2}] = [k][K]$$

$$[k] = [ML^2T^{-2}K^{-1}]$$

Q,

$$\text{Use, } F = \eta A\frac{dv}{dx}$$

$$[\eta] = \frac{[F]}{[A]}\left[\frac{dx}{dv}\right] = \frac{[MLT^{-2}]}{[L^2]}\left[\frac{L}{LT^{-1}}\right] = [ML^{-1}T^{-1}]$$

R,

$$\text{Use, } E = h\nu$$

$$[h] = \frac{[E]}{[\nu]} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

S,

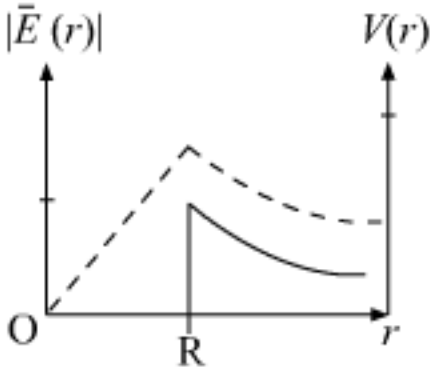
$$\text{Use, } \frac{dQ}{dt} = \frac{kA\Delta T}{l}$$

$$[k] = \frac{[ML^2T^{-3}][L]}{[L^2][K]} = [MLT^{-3}K^{-1}]$$

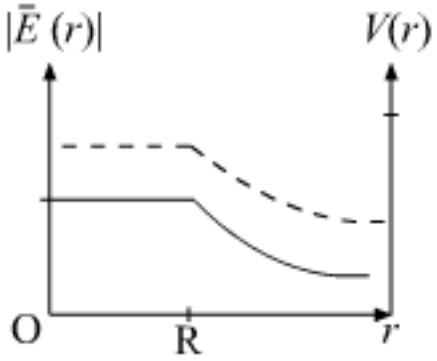
Q3. IIT JEE 2012 Paper - 1

Consider a thin spherical shell of radius *R* with its center at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance *r* from the centre, is best represented by which graph?

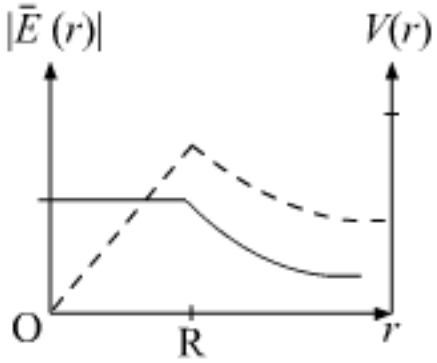
(a)



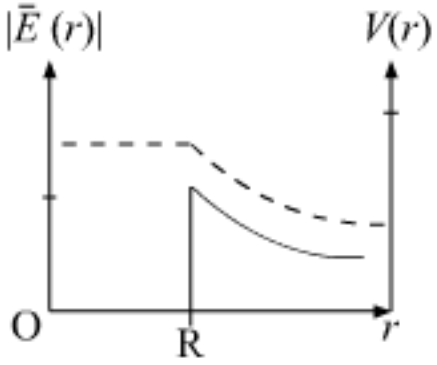
(b)



(c)

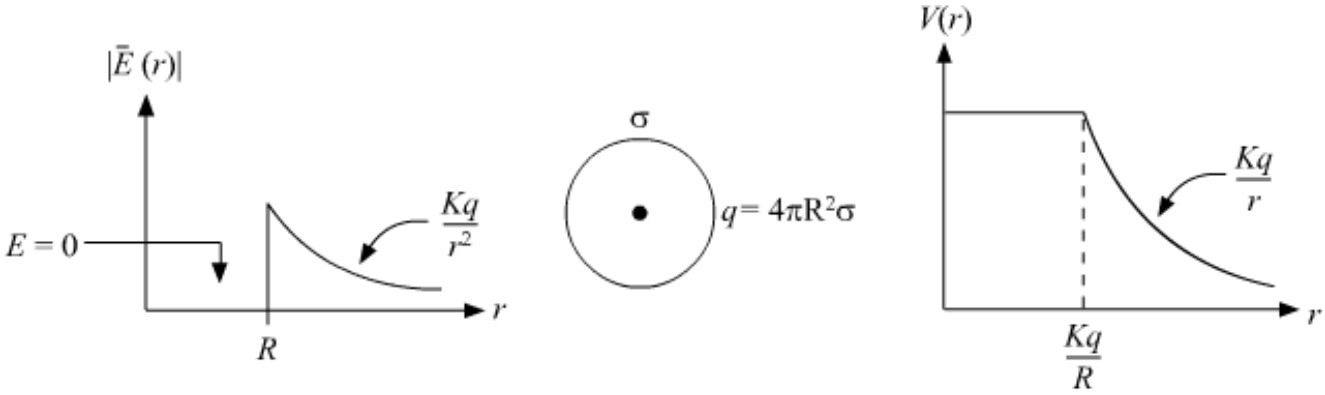


(d)



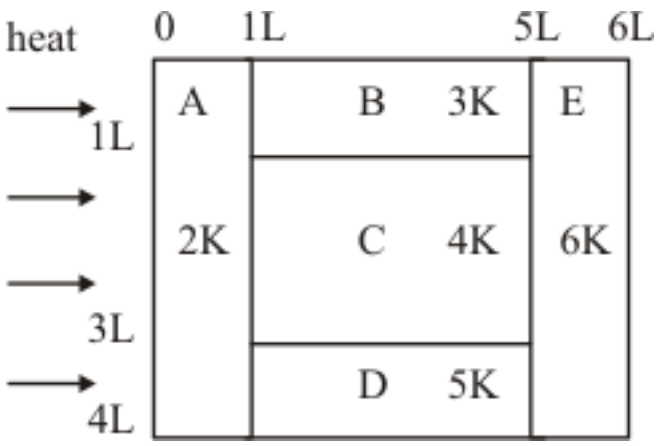
Solution:

For a spherical shell



Q4. IIT JEE 2011 Paper - 1

A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



(a) Heat flow through A and E slabs are same

(b) Heat flow through slab E is maximum

(c) Temperature difference across slab E is smallest

(d) Heat flow through C = heat flow through B + heat flow through D

Correct Answer: Option C

A, (BCD), E are in series. Thus

$H_A = H_E$

$= H_B + H_C + H_D$

This gives correct option (A) and (B).

For parallel slabs B, C and D we have $H \propto KA$.

$\therefore H_B : H_C : H_D = 3KL : 8KL : 5KL$

This gives correct option (D).

For series, $K\Delta\theta$ is same. Since K_E is largest, $\Delta\theta_E$ is least. This gives correct option (C).

Q5. IIT JEE 2011 paper - 1

Four solid spheres each of diameter $\sqrt{5}cm$ and mass 0.5 kg are placed with their centres at the corner of side 4cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4} kg - m^2$, then N is

Solution:

The moment of the inertia is given by the relation given below.

$$I = 2\left[\left\{\frac{2}{5}mr^2 + m\left(\frac{a}{\sqrt{2}}\right)^2\right\} + \left\{\frac{2}{5}mr^2\right\}\right]$$

$$= 9\times10^{-4}\,kg - m^2$$

Chemistry>

Q1. IIT JEE 2012 Paper - 2

$NiCl_2\{P(C_2H_5)_2(C_6H_5)\}_2$ exhibits temperature dependent magnetic behavior (paramagnetic /diamagnetic). The coordination geometries of Ni^{2+} in the paramagnetic and diamagnetic states are respectively

(A) Tetrahedral and tetrahedral
(B) Square planar and square planar
(C) Tetrahedral and square planar
(D) Square planar and tetrahedral

Solution:

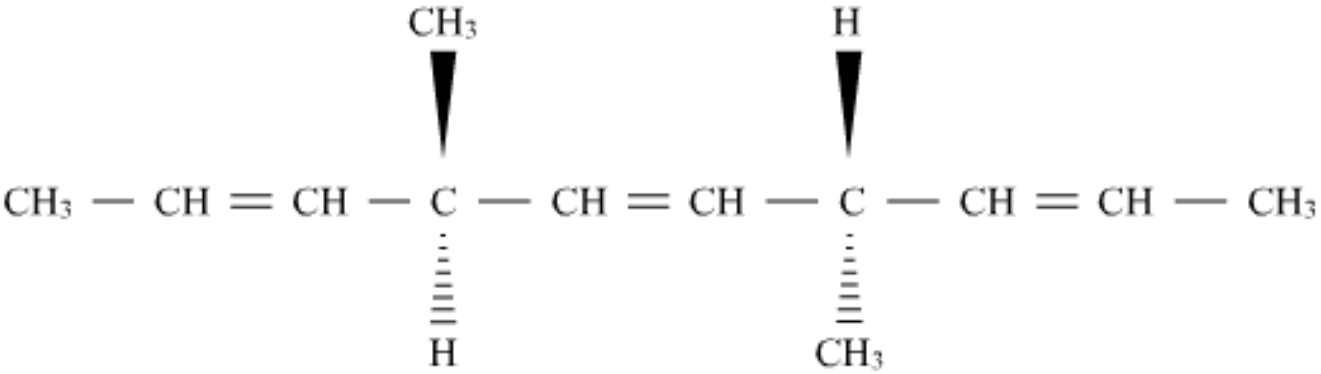
The electronic configuration of $Ni^{+2} = [Ar]^{18}\, 3d^8 4s^0$

If it is paramagnetic it should be sp^3, high spin & tetrahedral in geometry.

If it is diamagnetic it should be dsp^2, low spin & square planar in geometry.

Q2. IIT JEE 2012 Paper - 2

The number of optically active products obtained from the complete ozonolysis of the given compound is

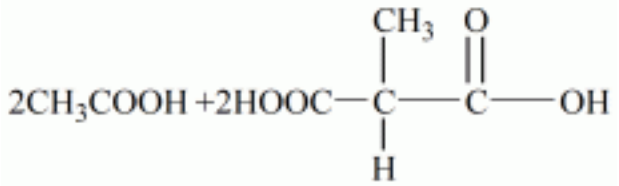


(A) 0
(B) 1
(C) 2
(D) 4

Solution:

Ozonolysis allows the rupturing of alkene double bonds of an unsaturated substance, followed by hydrolysis of the resulting ozonide − produced oxygenated fragments that are capable of forming reacting differentiable crystalline derivatives.

The products of ozonlysis are:



Therefore, number of optically active product will be four, two of each of the products

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Q3. IIT JEE 2012 Paper – 2

The electrochemical cell shown below is a concentration cell. $M|M^{+2}$ (Saturated solution of a sparingly soluble salt, MX_2) $\|M^{+2}\ (0.001\ \text{mol dm}^{-3})|M$ The emf of the cell depends on the difference in concentration of M^{2+} ions at the two electrodes. The emf of the cell at 298 K is 0.059 V.

The solubility product $\left(K_{sp}\ ;\text{mol}^3\ \text{dm}^{-9}\right)$ of MX_2 at 298 K based on the information available for the given concentration cell is (take $2.303\times R\times 298/F=0.059V$)

(A) 1×10^{-15}

(B) 4×10^{-15}

(C) 1×10^{-12}

(D) 4×10^{-12}

Correct Answer: Option B

Cell reaction for the cell is $M\left|M^{+2}(aq)\right\|M^{+2}\left|(aq)M\right.$

Reaction at Anode: $M \rightarrow M^{+2}(aq)+2e^{-}$

Reaction at cathode: $M^{+2}(aq)+2e^{-} \rightarrow M$

$$E^{0}_{cell}=0-\left(\frac{0.059}{2}\right)\log\left[M^{+2}\right]+\left(\frac{0.059}{2}\right)\log 10^{-3}$$

$$=\left(\frac{0.059}{2}\right)\log\left(\frac{10^{-3}}{M^{+2}}\right)$$

$$-2=\log\left\{\frac{M^{+2}}{10^{-3}}\right\}$$

$$10^{-2}\times 10^{-3}=M^{+2}=\text{solubility}=s$$

$$K_{sp}=4s^{3}$$

$$=4\times\left(10^{-5}\right)^{3}$$

$$=4\times 10^{-15}$$

Q4. IIT JEE 2012 Paper - 1

An organic compound undergoes first-order decomposition. The time taken for its decomposition to 1/8 and 1/10 of its concentration are $t_{1/8}$ and $t_{1/10}$ respectively.

What is the value of $\frac{\left[t_{1/8}\right]}{\left[t_{1/10}\right]}\times 10\,?\left(\text{take } \log_{10}2=0.3\right)$

Solution:

The integrated first order rate law is,

$$\ln[A] = -kt + \ln[A]_0$$

$$\frac{[Kt_{1/8}]}{[Kt_{1/10}]} = \frac{\ln[A]_0 - \ln[A]_{1/8}}{\ln[A]_0 - \ln[A]_{1/10}}$$

$$= \frac{\log 8}{\log 10}$$

$$= \frac{3\log 2}{\log 10}$$

We know that value of log10 = 1

In the given log value, log2 = 0.3

Thus,

$$\frac{[t_{1/8}]}{[t_{1/10}]} \times 10 = 3 \times (0.3) \times 10$$

$$= 9$$

Hence the value of $\frac{[t_{1/8}]}{[t_{1/10}]} \times 10$ is $\boxed{9}$
