Formula Sheet

Unit: Atoms and Nuclei

1. Rutherford's α – particle scattering

(a) $N(\theta) \propto \text{cosec}^4\ (\theta/2)$

(b) Impact parameter, $b = \frac{(Ze^2)\cot(\theta/2)}{(4\pi\varepsilon_0)E}$, (where $E = \frac{1}{2}mu^2 = KE$ of the α – particle)

2. Distance of closest approach:

$r_0 = \frac{2Ze^2}{(4\pi\varepsilon_0)E}$, (where $E = \frac{1}{2}mu^2 = KE$ of the α – particle)

3. Bohr's atomic model

(a) $L = mvr = \frac{nh}{2\pi}$

(b) $hv = E_i = E_f = \frac{hc}{\lambda}$

(c) Radius of nth orbit :

$r_n \propto \frac{n^2}{Z}$, $r_n = \frac{n^2}{Z}\left(\frac{h^2}{4\pi^2 m\, ke^2}\right)$

(d) Velocity of electron in nth orbit :

$v_n = \frac{Z}{n}\left(\frac{c}{137}\right) = \frac{Z}{n}\alpha c$ (where $\alpha = \frac{2\pi Ke^2}{ch} = \frac{1}{137}$ = fine structures constant)

4. Total energy of electron:

(a) Potential energy, $U = -(kZe^2/r)$

(b)Kinetic energy, $K = \frac{1}{2}mv^2 = (kZe^2/2r)$

(c) Total energy, $E = K + U = -(kZe^2/2r) = (U/2) = -K$

(d) $K = -(U/2)$ or $U = 2K = 2E$

(e) $E_n = \frac{13.6 Z^2}{n^2} eV = \frac{Z^2}{n^2}\left(\frac{2\pi^2 m k^2 e^4}{h^2}\right) = \frac{2.18 \times 10^{-18} Z^2}{n^2} J$

(f) Series formula (wave number $\bar{v} = 1/\lambda$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ were } R = \frac{2\pi^2 m k^2 e^4}{c h^3} = 1.097 \times 10^7 m^{-1}$$

(g) Series formula for H – atom

(i) Lyman series : $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right), n = 2, 3, 4, ... \infty$

(ii) Balmer series : $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), n = 3, 4, 5, ... \infty$

(iii) Paschen series : $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), n = 4, 5, 6, ... \infty$

(iv) Bracket series : $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), n = 5, 6, 7, ... \infty$

(v) P – fund series : $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), n = 6, 7, 8, ... \infty$

5. Number of emission lines from excited state:

$n = n(n - 1)/2$

6. Time period of revolution:

$T_n \propto (n^3/Z^2)$

7. Frequency of revolution:

$v_n \propto (Z^2/n^3)$

8. Current due to orbital motion:

$I_n \propto (Z^2/n^3)$

9. Magnetic field at nucleus due to orbital motion of electron

$B_n \propto (Z^3/n^5)$

10. Magnetic moment:

(a) $M_n = (eL/2m) = (nhe/4\pi m)$;

(b) $M_1 = (eh/4\pi m) = \mu_B$ = Bohr Magneton = 9.27×10^{-24} Am^2

11. Magnitude of angular momentum :

$L = \sqrt{[\ell(\ell + 1)]}\,(h/2\pi)$

12. Angle of angular momentum vector from z – axis

(a) $\cos\theta = [m_l \sqrt{\{\ell(\ell + 1)\}}]$

(b) the least angle is for $m_\ell = \ell$ i.e. $\cos\theta_{min} = [\ell/\sqrt{\{\ell(\ell + 1)\}}]$

13. Magnitude of spin angular momentum

$$S=\sqrt{[s(s+1)]}(h/2\pi)=\frac{\sqrt{3}}{2}(h/2\pi)$$

X – RAYS

1. Continuous X – rays:

(a) $v_{max} = (eV/h)$

(b) $\lambda_{min} = (hc/eV) = (12400/V)$ Å

2. Characterictic X – rays :

(a) $\lambda_{K\alpha} < \lambda_{L\alpha} < \lambda_{M\alpha}$

(b) $V_{K\alpha} > V_{L\alpha} > V_{M\alpha}$

3. Frequency of Kα line:

$$v(K_\alpha) = \frac{3\,cR}{4}(Z-1)^2 = 2.47 \times 10^{15}(Z-1)^2$$

4. Wavelength of Kα line:

$\lambda(K_\alpha) = [4/3R(Z-1)^2] = [1216/(Z-1)^2]$ Å

5. Energy of K_α X-ray photon:

$E(K_\alpha) = 10.2\,(Z-1)^2$ eV

6. Mosley's law:

(a) $v = a\,(Z-b)^2$, were $a = (3cR/4) = 2.47 \times 10^5$ Hz

(b) For K_α line, b = 1;

(c) $\sqrt{v}\ \alpha\ Z$

7. Bragg's law:
$2d \sin\theta = n\lambda$

8. Absorption formula:

$I = I_0\, e^{-\mu x}$

9. Half – value thickness:

$x_{1/2} = (0.693/\mu)$

MATTER WAVES

1. For photons:

(a) $E = h\nu = (hc/\lambda)$;

(b) $p = (h\nu/c) = (E/c) = (h/\lambda)$;

(c) $m = (E/c^2) = (h\nu/c^2) = h/c\lambda$

(d) rest mass = 0, charge = 0, spin = 1 $(h/2\pi)$

2. Matter waves:

(a) de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$ $\left[\because E = \frac{1}{2}mv^2 = qV\right]$

(i) For electron $\lambda_e = \frac{12.27}{\sqrt{V}}\,\mathring{A}$

(ii) For proton $\lambda_p = \frac{0.286}{\sqrt{V}}\,\mathring{A}$

(iii) For alpha particle $\lambda_\alpha = \frac{0.101}{\sqrt{V}}\,\mathring{A}$

(b) For particle at temperature T: $\lambda = \frac{h}{\sqrt{3mKT}}\left(E = \frac{3}{2}kT\right)$

(c) The wavelength of electron accelerated by potential difference of V volts is: $\lambda_e = \frac{12.27}{\sqrt{V}}\,\mathring{A}$

(d) Condition for obtaining stable orbit $2\pi r_n = n\lambda$

(e) The phase velocity of a de Broglie wave of wavelength λ and frequency ν is:

$$v_p = \nu\lambda = \frac{E}{h} \times \frac{h}{mv} = \frac{mc^2}{h} \times \frac{h}{mv} = \frac{c^2}{v} \text{ i.e. } v_p > c$$

(f) Group velocity, $v_g = (d\omega/dk)$. It is found that group velocity is equal to particle velocity i.e., $v_g = v$

RADIOACTIVITY

1. Decay law:

(a) $(dN/dt) = -\lambda N$;

(b) $N = N_0 e^{-\lambda t}$

(c) $(N/N_0) = (1/2)^{t/T}$

2. Half life and decay constant:

(a) $\lambda = -\frac{(dN/dt)}{N}$;

(b) $\lambda T = \log_e 2$ or $T = (0.693/\lambda)$ or $\lambda = (0.693/T)$

3. Mean life:

(a) $\tau = (1/\lambda)$ or $\lambda = (1/\tau)$;

(b) $T = 0.693\tau$ or $\tau = 1.443\ T$

4. Activity:

(a) $R = |dN/dt|$;

(b) $R = \lambda N$;

(c) $R = R_0 e^{-\lambda t}$;

(d) $(R/R_0) = (1/2)^{t/T}$;

(e) 1 Becquerel = 1 dps;

(f) 1 curie = 1 ci = 3.7×10^{10} dps;

(g) 1 Rutherford = 1 Rd = 10^6 Rd = 10^6 dps

5. Decay of active mass:

(a) $m = m_0 e^{-\lambda t}$;

(b) $(m/m_0) = (1/2)^{t/T}$;

(c) $N = \frac{6.023 \times 10^{23} \times m}{A}$

6. Radioactive equilibrium:

$N_A \lambda_A = N_B \lambda_B$

7. Decay constant for two channels:

(a) $\lambda = \lambda_1 + \lambda_2$;

(b) $T = \frac{T_1 T_2}{T_1 + T_2}$

8. Gamma Intensity absorption:

(a) $I = I_0 e^{-\mu x}$;

(b) Half value thickness, $x_{1/2} = (0.693/\mu)$

NUCLEAR PHYSICS

1. Atomic mass unit:

(a) 1 amu = 1.66×10^{-27} kg;

(b) 1 amu ≡ 1u ≡ 931.5 MeV

2. Properties of nucleus:

(a) Radius: $R = R_0A^{1/3}$ where R_0 =1.2 fermi

(b) Volume: $V \alpha A \left[\because V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A\right]$

(c) Density: $\rho = 2.4 \times 10^{17}$ Kg/m^3 (ρ is independent of A)

3. Mass defect:

$\Delta M = Zm_p + (A-Z)\, m_n - M$

4. Packing fraction:

$f = \Delta/A$ = mass excess per nucleon [$\Delta = -\Delta M$ = mass excess]

5. Binding energy:

$\Delta E = BE = (\Delta M)c^2$

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